Real-time optimal control of aerospace systems with state-control constraints and delays

Bruno HÉRISSÉ

ONERA (DTIS/NGPA)

Outline

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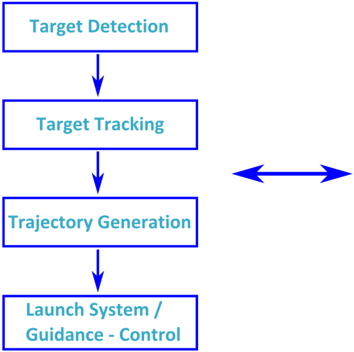
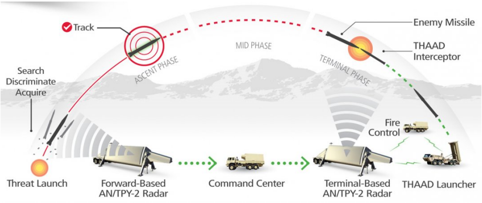
1. Introduction
   - Optimal Control & Applications at ONERA
   - Real-time Optimal Control
2. Optimal Interception Problem (R. Bonalli, B. Hérissé, E. Trélat)
   - Geometrical Methods to handle mixed constraints
   - Optimal Interception Problem
   - Optimal Interception Problem with Delays
3. Return trajectory of Reusable Launch Vehicles (CNES-ONERA project)
4. Conclusion

Interception Problem

- **Objective** : Intercept the target, need high precision and high terminal velocity.
- **Difficulty** : Missiles can fly at high altitude (20-30 km), difficult to control with aerodynamic actuators due to altitude dependance of air density.





THAAD Interceptor [https://www.mda.mil]

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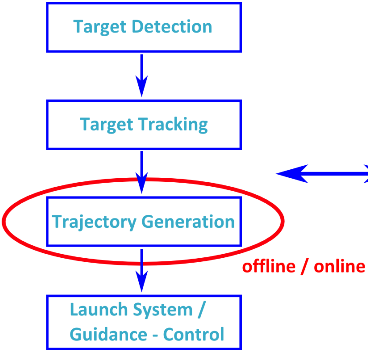

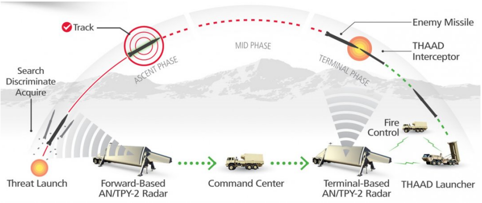

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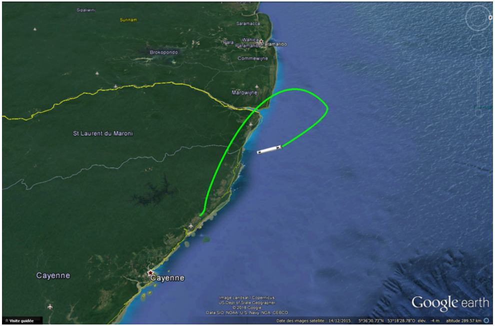
Reusable Launch Vehicles

- **Objective** : Return and landing of the first stage of space launchers.
- **Difficulty** : Highly constrained problem (aerodynamic and safety constraints), may need re-ignition of rocket engines.



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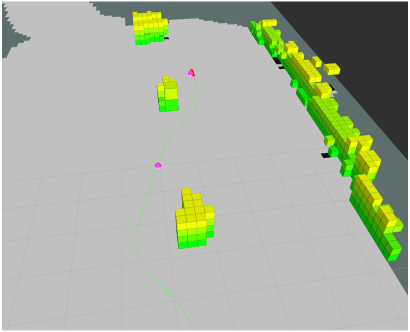
Motion Planning for Unmanned Aerial Vehicles

- **Objective** : Motion planning of Aerial Robots in cluttered environments.
- **Difficulty** : Dynamic environments (moving obstacles, etc.), obstacle modeling.



Real-time Optimal Control

Challenge

Compute optimal trajectories in **real time**, by using **embedded computers**, to make the vehicle adapt its trajectory to **changes of the scenario**.

1. Explicit Feedback Laws and Direct Methods:

Pros:
- Easy to implement
- Robustness

Cons:
- Lack of precision
- Expensive or sub-optimal

2. **Indirect Methods:**

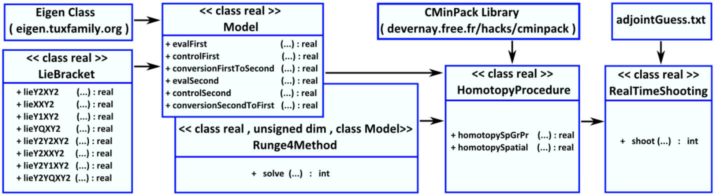
Pros:
- High precision
- **Fast convergence**

Cons:
- Complex analysis
- **Hard to initialize**

Library for solving real-time optimal control problems

SOCP (Shooting for Optimal Control Problems)

- **Indirect methods** for high precision.



Library for solving real-time optimal control problems

SOCP (Shooting for Optimal Control Problems)

- **Indirect methods** for high precision.
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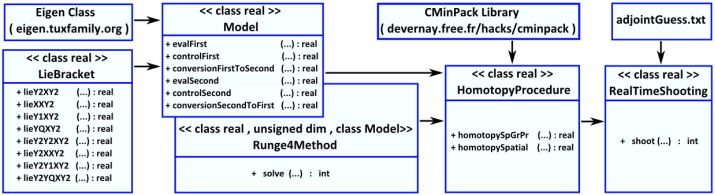

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SOCP (Shooting for Optimal Control Problems)

- **Indirect methods** for high precision.
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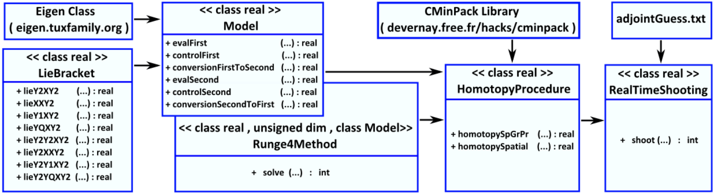
- **Multiple shooting** for numerical robustness.



Library for solving real-time optimal control problems

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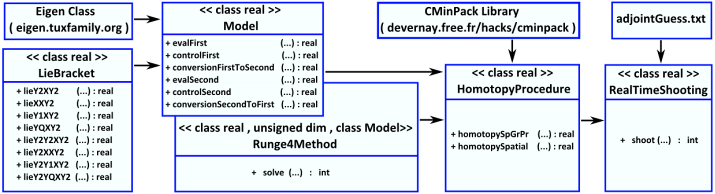
- **Parallel computing** for computation time.



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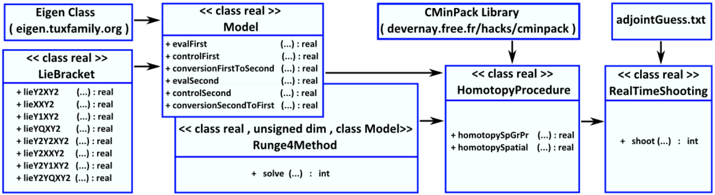
Library for solving real-time optimal control problems

SOCP (Shooting for Optimal Control Problems)

- **Indirect methods** for high precision.
- **homotopy methods** for initialization problems.
- **Multiple shooting** for numerical robustness.
- **Parallel computing** for computation time.
- **C++ library** for best performance and embedded solutions.



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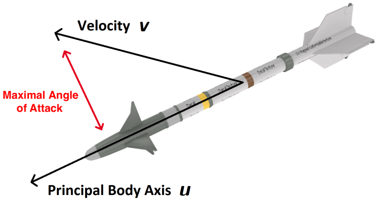
3 Return trajectory of Reusable Launch Vehicles (CNES-ONERA project)

4 Conclusion

General Optimal Guidance Problem (GOGP)

General Optimal Guidance Problem (GOGP)

$$
\begin{cases}
\min \quad g(t_f, \boldsymbol{r}(t_f), \boldsymbol{v}(t_f)) \\
\dot{\boldsymbol{r}} = \boldsymbol{v} \quad , \quad (\boldsymbol{r}, \boldsymbol{v}) \in \mathbb{R}^6 \setminus \{0\} \\
\dot{\boldsymbol{v}} = \boldsymbol{f}_{Total} = \boldsymbol{f}_{Aero}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) + \boldsymbol{f}_{ThGr}(t, \boldsymbol{r}, \boldsymbol{u}) \\
(\boldsymbol{r}, \boldsymbol{v})(0) = (\boldsymbol{r}_0, \boldsymbol{v}_0) \quad , \quad (\boldsymbol{r}, \boldsymbol{v})(t_f) \in M_f \\
\boldsymbol{u}(t) \in S^2 \quad , \quad c(\boldsymbol{v}(t), \boldsymbol{u}(t)) \leq 0
\end{cases}
$$



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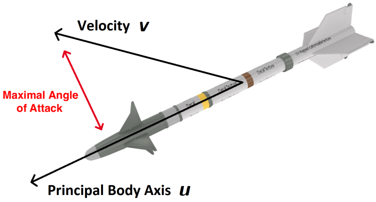

Detailed Quantities

- $\boldsymbol{f}_{Aero} = \boldsymbol{f}_{Drag} + \boldsymbol{f}_{Lift}$, $\boldsymbol{f}_{ThGr} = \boldsymbol{f}_{Thrust} + \boldsymbol{f}_{Gravity}$
- $c = (c_1, c_2) = \left(-\frac{\boldsymbol{v} \cdot \boldsymbol{u}}{\|\boldsymbol{v}\|} \; , \; \left(\frac{\|\boldsymbol{u} \wedge \boldsymbol{v}\|}{\|\boldsymbol{v}\|} \right)^2 - \sin^2 \alpha_{max} \right)$

Maximum Principle Applied to (GOGP): Related Issues

General Optimal Guidance Problem (GOGP)

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\begin{cases}
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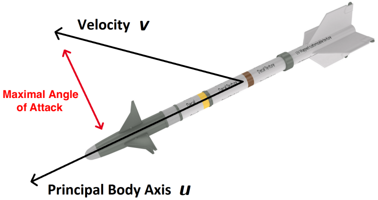


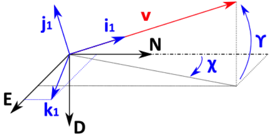
Pontryagin Maximum Principle for Problems with Mixed Constraints

There exist an adjoint vector $\boldsymbol{p}(\cdot)$ and an additional multiplier $\mu(\cdot)$, such that

$$
\dot{\boldsymbol{p}}(t) = -\boldsymbol{p}(t) \cdot \frac{\partial \boldsymbol{f}_{Total}}{\partial(\boldsymbol{r}, \boldsymbol{v})}(t, \boldsymbol{r}(t), \boldsymbol{v}(t), \boldsymbol{u}(t)) - \mu(t) \cdot \frac{\partial c}{\partial(\boldsymbol{r}, \boldsymbol{v})}(\boldsymbol{v}(t), \boldsymbol{u}(t))
$$

Local Evaluation of (**GOGP**): Euler Coordinates

Exploit local Euler coordinates by means of the **path-azimuth local frame**.



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Local Evaluation of (GOGP): Euler Coordinates

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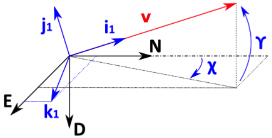


Local Dynamics

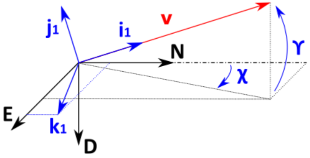
$$\begin{cases} \dot{r} = v\sin\gamma \; , \; \dot{L} = \frac{v}{r}\cos\gamma\cos\chi \; , \; \dot{\ell} = \frac{v}{r}\frac{\cos\gamma\sin\chi}{\cos L} \\ \dot{v} = \frac{f_T}{m}\mathbf{w}_1 - \left(d + \eta c_m\left(\mathbf{w}_2^2 + \mathbf{w}_3^2\right)\right)v^2 - g\sin\gamma \\ \dot{\gamma} = \omega\mathbf{w}_2 + \left(\frac{v}{r} - \frac{g}{v}\right)\cos\gamma \\ \dot{\chi} = \frac{\omega}{\cos\gamma}\mathbf{w}_3 + \frac{v}{r}\cos\gamma\sin\chi\tan L \end{cases}$$

$$\omega(t) = \frac{f_T(t)}{m(t)v(t)} + v(t)c_m(t) > 0$$

Crucial Properties

- Well-known among engineers.
- Intuition is easier.
- The mixed constraint does not depend on v $\rightarrow$ **pure control constraint**.
- **Singularity for** $\gamma = \pi/2$.

Local Evaluation of (**GOGP**): Euler Coordinates



Projection onto Frame $(\boldsymbol{i}_1, \boldsymbol{j}_1, \boldsymbol{k}_1)$

$$\boldsymbol{u} = w_1 \boldsymbol{i}_1(\gamma, \chi) + w_2 \boldsymbol{j}_1(\gamma, \chi) + w_3 \boldsymbol{k}_1(\gamma, \chi)$$

$$c(w_1, w_2, w_3) = \Big(-w_1 \;,\; w_2^2 + w_3^2 - \sin^2 \alpha_{\max} \Big)$$

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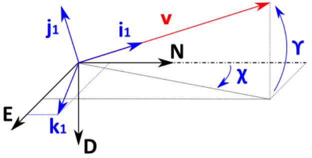
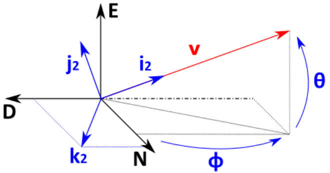
Local Evaluation of (GOGP): Euler Coordinates



$(\theta, \phi) \leftrightarrow F(\gamma, \chi)$



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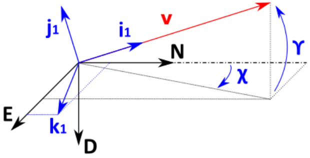
$$c(w_1, w_2, w_3) = \Big(-w_1 \ , \ w_2^2 + w_3^2 - \sin^2 \alpha_{max} \Big)$$

Projection onto Frame $(\boldsymbol{i}_2, \boldsymbol{j}_2, \boldsymbol{k}_2)$

$$\boldsymbol{u} = z_1 \boldsymbol{i}_2(\theta, \phi) + z_2 \boldsymbol{j}_2(\theta, \phi) + z_3 \boldsymbol{k}_2(\theta, \phi)$$

$$c(z_1, z_2, z_3) = \Big(-z_1 \ , \ z_2^2 + z_3^2 - \sin^2 \alpha_{max} \Big)$$

Local Evaluation of (GOGP): Euler Coordinates



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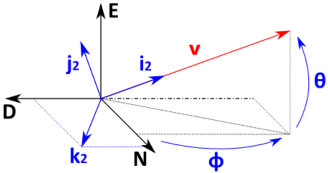

Projection onto Frame $(\boldsymbol{i}_1, \boldsymbol{j}_1, \boldsymbol{k}_1)$

$$\boldsymbol{u} = w_1 \boldsymbol{i}_1(\gamma,\chi) + w_2 \boldsymbol{j}_1(\gamma,\chi) + w_3 \boldsymbol{k}_1(\gamma,\chi)$$

$$c(w_1, w_2, w_3) = \Big(-w_1 \ , \ w_2^2 + w_3^2 - \sin^2 \alpha_{\max} \Big)$$

Projection onto Frame $(\boldsymbol{i}_2, \boldsymbol{j}_2, \boldsymbol{k}_2)$

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$(\mathbf{GOGP})_1$

$$\begin{cases} \min \ g(t_f, \boldsymbol{r}(t_f), \boldsymbol{v}(t_f)) \ , \ (\boldsymbol{r},\boldsymbol{v}) \in V_1 \\ \dot{\boldsymbol{r}} = \boldsymbol{v} \ , \quad \dot{\boldsymbol{v}} = \boldsymbol{f}^1_{Total}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{w}) \\ c(\boldsymbol{w}(t)) = c(w_1(t), w_2(t), w_3(t)) \le 0 \end{cases}$$

$(\mathbf{GOGP})_2$

$$\begin{cases} \min \ g(t_f, \boldsymbol{r}(t_f), \boldsymbol{v}(t_f)) \ , \ (\boldsymbol{r},\boldsymbol{v}) \in V_2 \\ \dot{\boldsymbol{r}} = \boldsymbol{v} \ , \quad \dot{\boldsymbol{v}} = \boldsymbol{f}^2_{Total}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{z}) \\ c(\boldsymbol{z}(t)) = c(z_1(t), z_2(t), z_3(t)) \le 0 \end{cases}$$

Local Maximum Principles for (GOGP): Euler Coordinates

$(\mathbf{PMP})_1$

If $(\boldsymbol{r}, \boldsymbol{v}, \boldsymbol{p}_1) \in T^* V_1$, then:

- $\boldsymbol{u} = F_1(\boldsymbol{r}, \boldsymbol{v}, \boldsymbol{w})$
- $\dot{\boldsymbol{p}}_1 = -\boldsymbol{p}_1 \cdot \dfrac{\partial \boldsymbol{f}^1_{Total}}{\partial(\boldsymbol{r}, \boldsymbol{v})}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{w})$

$(\mathbf{PMP})_2$

If $(\boldsymbol{r}, \boldsymbol{v}, \boldsymbol{p}_2) \in T^* V_2$, then:

- $\boldsymbol{u} = F_2(\boldsymbol{r}, \boldsymbol{v}, \boldsymbol{z})$
- $\dot{\boldsymbol{p}}_2 = -\boldsymbol{p}_2 \cdot \dfrac{\partial \boldsymbol{f}^2_{Total}}{\partial(\boldsymbol{r}, \boldsymbol{v})}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{z})$

Local Maximum Principles for (GOGP): Euler Coordinates

(PMP)$_1$

If $(\boldsymbol{r}, \boldsymbol{v}, \boldsymbol{p}_1) \in T^* V_1$, then:

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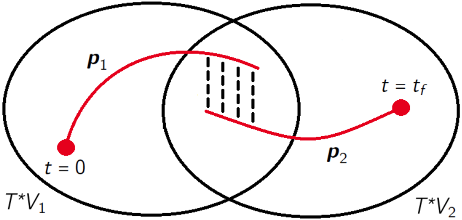
(PMP)$_2$

If $(\boldsymbol{r}, \boldsymbol{v}, \boldsymbol{p}_2) \in T^* V_2$, then:

- $\boldsymbol{u} = F_2(\boldsymbol{r}, \boldsymbol{v}, \boldsymbol{z})$
- $\dot{\boldsymbol{p}}_2 = -\boldsymbol{p}_2 \cdot \dfrac{\partial \boldsymbol{f}^2_{Total}}{\partial(\boldsymbol{r}, \boldsymbol{v})}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{z})$

Difficulty

Since $\boldsymbol{w} \neq \boldsymbol{z}$, adjoints $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ may not be the same mappings within $T^* V_1 \cap T^* V_2$...
→ In our case, $\boldsymbol{p}_1 = \boldsymbol{p} = \boldsymbol{p}_2$!



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Optimal Interception Problem (OIP)

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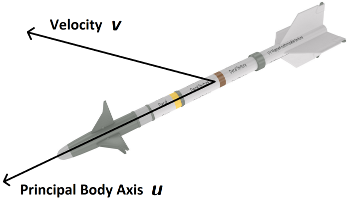
$$\begin{cases} \min \quad C(t_f, \boldsymbol{u}) \\ \dot{\boldsymbol{r}} = \boldsymbol{v} \quad , \quad (\boldsymbol{r}, \boldsymbol{v}) \in \mathbb{R}^6 \setminus \{0\} \\ \dot{\boldsymbol{v}} = \boldsymbol{f}_{Aero}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) + \boldsymbol{f}_{ThGr}(t, \boldsymbol{r}, \boldsymbol{u}) \\ (\boldsymbol{r}, \boldsymbol{v})(0) = (\boldsymbol{r}_0, \boldsymbol{v}_0) \quad , \quad (\boldsymbol{r}, \boldsymbol{v})(t_f) \in M_f \\ \boldsymbol{u}(t) \in S^2 \quad , \quad c(\boldsymbol{v}(t), \boldsymbol{u}(t)) \leq 0 \end{cases}$$



Specific Cost and Final Conditions

- $C(t_f, \boldsymbol{u}) = C_{t_f} t_f - \|\boldsymbol{v}(t_f)\|^2 \quad , \quad C_{t_f} \in \{0, 1\}$
- $M_f = \left\{ (\boldsymbol{r}, \boldsymbol{v}) \in \mathbb{R}^6 \setminus \{0\} \mid \boldsymbol{r} = \boldsymbol{r}_f \ , \ \boldsymbol{v}/\|\boldsymbol{v}\| = \boldsymbol{v}_f/\|\boldsymbol{v}_f\| \right\}$

Numerical Strategy to Solve (OIP)

Strategy

- Choose a Problem of Order Zero $(\mathbf{OIP})_0$.
- Build a converging homotopy scheme.

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Numerical Strategy to Solve (**OIP**)

Strategy

- Choose a Problem of Order Zero $(\mathbf{OIP})_0$.
- Build a converging homotopy scheme.

Optimal Interception Problem of Order Zero $(\mathbf{OIP})_0$

$$
\begin{cases}
\min \quad -\|\boldsymbol{v}(t_f)\|^2 \\
\dot{\boldsymbol{r}} = \boldsymbol{v} \quad , \quad (\boldsymbol{r}, \boldsymbol{v}) \in \mathbb{R}^6 \setminus \{0\} \\
\dot{\boldsymbol{v}} = \boldsymbol{f}_{Aero}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) \\
(\boldsymbol{r}, \boldsymbol{v})(0) = (\boldsymbol{r}_0, \boldsymbol{v}_0) \ , \ (\boldsymbol{r}, \boldsymbol{v})(t_f) \in M_f^0 \\
\boldsymbol{u}(t) \in S^2 \quad , \quad c(\boldsymbol{v}(t), \boldsymbol{u}(t)) \in \mathbb{R}
\end{cases}
$$

Modifications

- Maximizing the final velocity
- Aerodynamical effects only
- Simplifying the original mission
- Removing mixed constraints

Crucial Feature

This problem respects the original geometric Lie configuration and can be **initialized explicitly** using realistic approximations.

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Homotopy Strategy to Solve (OIP)

Two-Parameters Homotopy Scheme

$$\begin{cases} g_{\lambda}(t_f, \boldsymbol{r}, \boldsymbol{v}) = -\|\boldsymbol{v}\|^2 + \lambda_1 \Big(g(t_f, \boldsymbol{r}, \boldsymbol{v}) + \|\boldsymbol{v}\|^2 \Big) \\ \boldsymbol{f}^{\lambda}_{Total}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) = \boldsymbol{f}_{Total}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) - (1 - \lambda_1) \left(\frac{\|\boldsymbol{T}(t)\|}{m} \boldsymbol{u} - g(\boldsymbol{r}) \frac{\boldsymbol{r}}{\|\boldsymbol{r}\|} \right) \\ M_f^{\lambda} = \lambda_2 M_f + (1 - \lambda_2) M_f^0 \end{cases}$$

$(\mathrm{OIP})_0$

$(\lambda_1 = 0, \lambda_2 = 0)$

$$\begin{cases} \min \quad -\|\boldsymbol{v}(t_f)\|^2 \\ \dot{\boldsymbol{r}} = \boldsymbol{v} \quad , \quad \boldsymbol{u}(t) \in S^2 \\ \dot{\boldsymbol{v}} = \boldsymbol{f}_{Aero}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) \\ (\boldsymbol{r}, \boldsymbol{v})(t_f) \in M_f^0 \\ c(\boldsymbol{v}(t), \boldsymbol{u}(t)) < 0 \end{cases}$$

Homotopy Strategy to Solve (OIP)

Two-Parameters Homotopy Scheme

$$\begin{cases} g_{\lambda}(t_f, \boldsymbol{r}, \boldsymbol{v}) = -\|\boldsymbol{v}\|^2 + \lambda_1\Big(g(t_f, \boldsymbol{r}, \boldsymbol{v}) + \|\boldsymbol{v}\|^2\Big) \\ \boldsymbol{f}^{\lambda}_{Total}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) = \boldsymbol{f}_{Total}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) - (1 - \lambda_1)\left(\frac{\|\boldsymbol{T}(t)\|}{m}\boldsymbol{u} - g(\boldsymbol{r})\frac{\boldsymbol{r}}{\|\boldsymbol{r}\|}\right) \\ M^{\lambda}_f = \lambda_2 M_f + (1 - \lambda_2) M^0_f \end{cases}$$

$(\text{OIP})_0$
$(\lambda_1 = 0, \lambda_2 = 0)$

$$\begin{cases} \min \quad -\|\boldsymbol{v}(t_f)\|^2 \\ \dot{\boldsymbol{r}} = \boldsymbol{v} \quad , \quad \boldsymbol{u}(t) \in S^2 \\ \dot{\boldsymbol{v}} = \boldsymbol{f}_{Aero}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) \\ (\boldsymbol{r}, \boldsymbol{v})(t_f) \in M^0_f \\ c(\boldsymbol{v}(t), \boldsymbol{u}(t)) < 0 \end{cases}$$

$\Longrightarrow$

$(\text{OIP})_{\text{Inter}}$
$(\lambda_1 = 1, \lambda_2 = 0)$

$$\begin{cases} \min \quad g(t_f, \boldsymbol{r}(t_f), \boldsymbol{v}(t_f)) \\ \dot{\boldsymbol{r}} = \boldsymbol{v} \quad , \quad \boldsymbol{u}(t) \in S^2 \\ \dot{\boldsymbol{v}} = \boldsymbol{f}_{Total}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) \\ (\boldsymbol{r}, \boldsymbol{v})(t_f) \in M^0_f \\ c(\boldsymbol{v}(t), \boldsymbol{u}(t)) \le 0 \end{cases}$$

Homotopy Strategy to Solve (OIP)

Two-Parameters Homotopy Scheme

$$\begin{cases} g_{\lambda}(t_f, \boldsymbol{r}, \boldsymbol{v}) = -\|\boldsymbol{v}\|^2 + \lambda_1\Big(g(t_f, \boldsymbol{r}, \boldsymbol{v}) + \|\boldsymbol{v}\|^2\Big) \\ \boldsymbol{f}^{\lambda}_{Total}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) = \boldsymbol{f}_{Total}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) - (1 - \lambda_1)\left(\frac{\|\boldsymbol{T}(t)\|}{m}\boldsymbol{u} - g(\boldsymbol{r})\frac{\boldsymbol{r}}{\|\boldsymbol{r}\|}\right) \\ M^{\lambda}_f = \lambda_2 M_f + (1 - \lambda_2) M^0_f \end{cases}$$

$(\text{OIP})_0$ $(\lambda_1 = 0, \lambda_2 = 0)$

$$\begin{cases} \min \quad -\|\boldsymbol{v}(t_f)\|^2 \\ \dot{\boldsymbol{r}} = \boldsymbol{v} \quad , \quad \boldsymbol{u}(t) \in S^2 \\ \dot{\boldsymbol{v}} = \boldsymbol{f}_{Aero}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) \\ (\boldsymbol{r}, \boldsymbol{v})(t_f) \in M^0_f \\ c(\boldsymbol{v}(t), \boldsymbol{u}(t)) < 0 \end{cases}$$

$\Longrightarrow$

$(\text{OIP})_{\text{Inter}}$ $(\lambda_1 = 1, \lambda_2 = 0)$

$$\begin{cases} \min \quad g(t_f, \boldsymbol{r}(t_f), \boldsymbol{v}(t_f)) \\ \dot{\boldsymbol{r}} = \boldsymbol{v} \quad , \quad \boldsymbol{u}(t) \in S^2 \\ \dot{\boldsymbol{v}} = \boldsymbol{f}_{Total}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) \\ (\boldsymbol{r}, \boldsymbol{v})(t_f) \in M^0_f \\ c(\boldsymbol{v}(t), \boldsymbol{u}(t)) \leq 0 \end{cases}$$

$\Longrightarrow$

(OIP) $(\lambda_1 = 1, \lambda_2 = 1)$

$$\begin{cases} \min \quad g(t_f, \boldsymbol{r}(t_f), \boldsymbol{v}(t_f)) \\ \dot{\boldsymbol{r}} = \boldsymbol{v} \quad , \quad \boldsymbol{u}(t) \in S^2 \\ \dot{\boldsymbol{v}} = \boldsymbol{f}_{Total}(t, \boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) \\ (\boldsymbol{r}, \boldsymbol{v})(t_f) \in M_f \\ c(\boldsymbol{v}(t), \boldsymbol{u}(t)) \leq 0 \end{cases}$$

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Computational Improvements for High Sensitivity

High Sensitivity to Initial Conditions

Computational times are not deterministic (different scenarios may take more or less computational time to converge).

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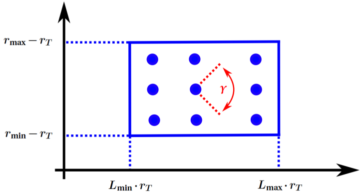
Initialization Grids

Compute optimal solutions for many feasible scenarios offline and store them into files.



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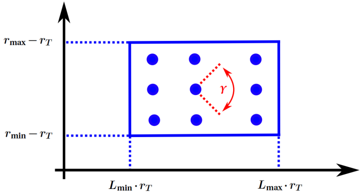
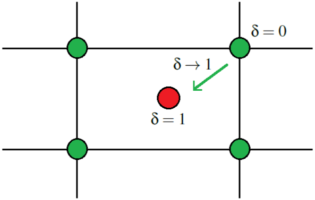
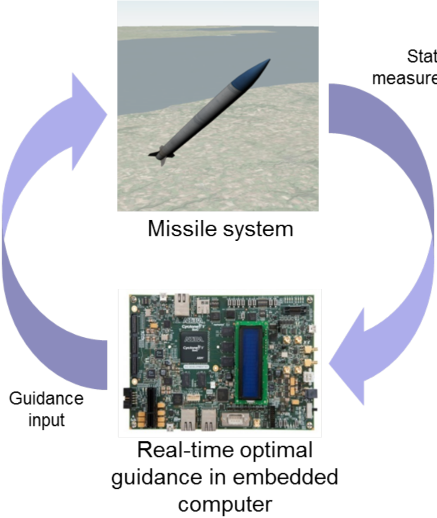


Efficient and Fast Computations

To solve any random scenario, choose among all the scenarios already solved the closest one and operate a spatial homotopy.



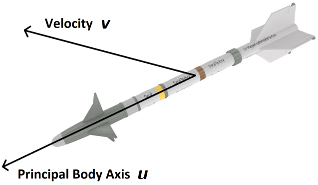
Hardware-In-the-Loop Simulation



Optimal Interception Problem with Delays $(\mathbf{OIP})_\tau$

Optimal Interception with Delays $(\mathbf{OIP})_\tau$

$$\begin{cases} \min \quad \int_0^{t_f} \|\omega(t)\|^2 \, dt - \|\mathbf{v}(t_f)\|^2 \\ \dot{\mathbf{r}} = \mathbf{v} \quad , \quad (\mathbf{r}, \mathbf{v}, \mathbf{u}) \in \mathbb{R}^6 \setminus \{0\} \times S^2 \\ \dot{\mathbf{v}} = \mathbf{f}_{Aero}(t, \mathbf{r}, \mathbf{v}, \mathbf{u}, \mathbf{u}(t - \tau)) + \mathbf{f}_{ThGr}(t, \mathbf{r}, \mathbf{u}) \\ \dot{\mathbf{u}} = \omega \quad , \quad \omega \in \mathbb{R}^3 \\ (\mathbf{r}, \mathbf{v}, \mathbf{u})(0) = (\mathbf{r}_0, \mathbf{v}_0, \mathbf{u}_0) \ , \ (\mathbf{r}, \mathbf{v}, \mathbf{u})(t_f) \in M_f \end{cases}$$



Challenge in Solving $(\mathbf{OIP})_\tau$

- Is numerical homotopy possible on the delay τ?
- Applying the Maximum Principle, a backward-forward equation of time appears.
  → A **global-in-time guess** of $p_\tau(\cdot)$ is needed.

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How Using Numerical Homotopy for Problems with Delays

Proposed Solution

Combining indirect methods with homotopy procedures on the delay.
→ Under appropriate assumptions, $p_\tau(\cdot)$ **is continuous (in strong topology)**.

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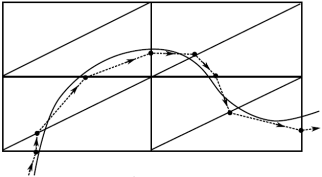
Delay as Deformation Parameter

Starting from the solution of the problem without delay, adding the contribution of the delay step by step to recover the solution of the original problem.



Parameter Deformation Delay Path

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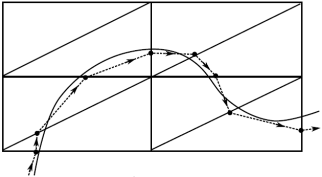

Parameter Deformation Delay Path

Solving Delayed Optimal Control Problems by Homotopy

$$\begin{cases} \dot{p}(t) = -p(t) \cdot \dfrac{\partial f}{\partial q_1}(q(t), q(t)) \\ \quad -p(t) \cdot \dfrac{\partial f}{\partial q_2}(q(t), q(t)) \end{cases} \quad \overset{k \uparrow}{\Longrightarrow} \quad \begin{cases} \dot{p}_{\tau_k}(t) = -p_{\tau_k}(t) \cdot \dfrac{\partial f}{\partial q_1}(q_{\tau_{k-1}}(t), q_{\tau_{k-1}}(t - \tau_k)) \\ \quad -\chi_{[0, t_f - \tau_k]}(t) p_{\tau_k}(t + \tau_k) \cdot \dfrac{\partial f}{\partial q_2}(q_{\tau_{k-1}}(t + \tau_k), q_{\tau_{k-1}}(t)) \end{cases}$$

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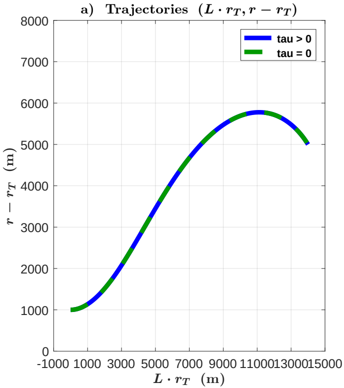
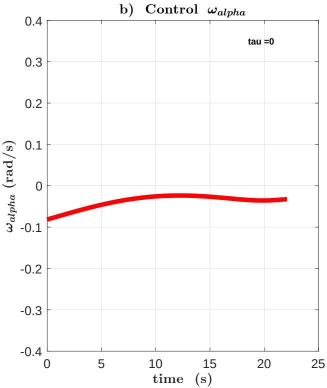
Numerical results for $(\mathbf{OIP})_\tau$





Outline

Return trajectory of Reusable Launch Vehicles

Objective (from stage separation to time before landing burn)

- Trajectory minimizing propellant consumption.
- Fixed final position and final velocity, free final time.
- Constraints on the angle-of-attack, dynamic pressure and thermal flux.

RLV model

$$
\begin{cases}
\min \quad - \quad m(t_f) \\
\dot{\boldsymbol{r}} = \boldsymbol{v} \quad , \quad (\boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) \in \mathbb{R}^6 \setminus \{0\} \times S^2 \\
\dot{\boldsymbol{v}} = \frac{L}{m} \boldsymbol{k}_a - \frac{D}{m} \frac{\boldsymbol{v}_a}{\|\boldsymbol{v}_a\|} - \boldsymbol{g} + \frac{T_m}{m} \gamma \boldsymbol{u} - 2\Omega \times \boldsymbol{v} - \Omega \times (\Omega \times \boldsymbol{r}) \\
\dot{m} = - q_m \gamma \\
(\boldsymbol{r}, \boldsymbol{v}, m)(0) = (\boldsymbol{r}_0, \boldsymbol{v}_0, m_0) \ , \ (\boldsymbol{r}, \boldsymbol{v})(t_f) = (\boldsymbol{r}_f, \boldsymbol{v}_f) \\
\alpha \leq \alpha_{\max} \quad \text{if} \quad \|r\| \leq r_{\text{ref}} \ , \quad Q \leq Q_{\max} \ , \quad \Phi \leq \Phi_{\max}
\end{cases}
$$

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RLV model (homotopy start with $\mu_x = 0$ except $\mu_{L_2} = 1$)

$$
\begin{cases}
\min \quad -\mu_{L_1} m(t_f) + \mu_{L_2} \int_0^{t_f} \gamma^2 \, dt \\
\dot{\boldsymbol{r}} = \boldsymbol{v} \quad , \quad (\boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) \in \mathbb{R}^6 \setminus \{0\} \times S^2 \\
\dot{\boldsymbol{v}} = \mu_L \frac{L}{m} \boldsymbol{k}_a - \mu_D \frac{D}{m} \frac{\boldsymbol{v}_a}{\|\boldsymbol{v}_a\|} - \mu_g \boldsymbol{g} + \frac{T_m}{m} \gamma \boldsymbol{u} - 2\Omega \times \boldsymbol{v} - \Omega \times (\Omega \times \boldsymbol{r}) \\
\dot{m} = -\mu_q q_m \gamma \\
(\boldsymbol{r}, \boldsymbol{v}, m)(0) = (\boldsymbol{r}_0, \boldsymbol{v}_0, m_0) \ , \ (\boldsymbol{r}, \boldsymbol{v})(t_f) = (\boldsymbol{r}_f, \boldsymbol{v}_f) \\
\mu_\alpha \alpha \leq \alpha_{\max} \quad \text{if} \quad \|r\| \leq r_{\text{ref}} \ , \ \mu_Q Q \leq Q_{\max} \ , \ \mu_\Phi \Phi \leq \Phi_{\max}
\end{cases}
$$

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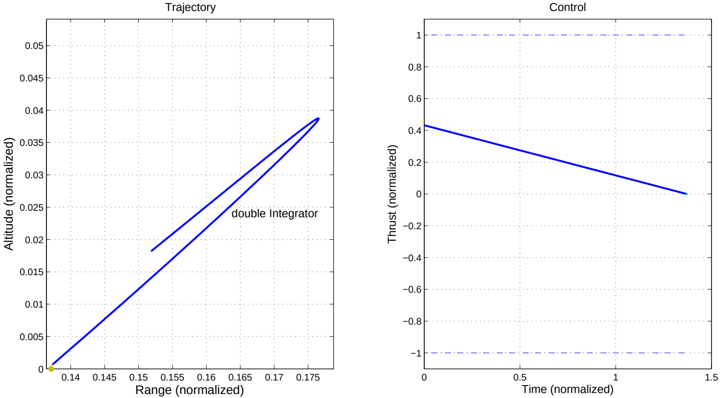
Return trajectory of RLV : numerical results



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Outline

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Conclusion

Conclusion

- An efficient **C++ library** implementing shooting methods.
- A general homotopy scheme for solving many complex aerospace problems.
- Real-time simulations show that **embedded optimal control is possible !**

Perspectives

- Feed the C++ library with other OCP and models.
- Improve robustness and computing time.
- Add features (more efficient numerical homotopy for delays, etc.).