Optimal control of Reusable Launch Vehicles: an indirect shooting approach

Prince EDORH, Elliot BRENDEL, Bruno HÉRISSÉ

ONERA (DTIS/NGPA)

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Outline

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Intercept Problem (Riccardo BONALLI's Ph.D. thesis)

- **Objective** : Intercept the target, need high precision and high terminal velocity.
- **Difficulty** : Missiles can fly at high altitude (20-30 km), difficult to control with aerodynamic actuators due to altitude dependance of air density.

Target Detection

↓

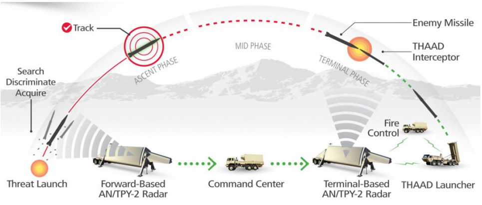
Target Tracking

↓

Trajectory Generation

↓

Launch System / Guidance - Control



THAAD Interceptor [https://www.mda.mil]

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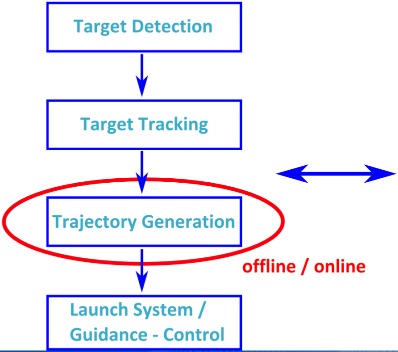

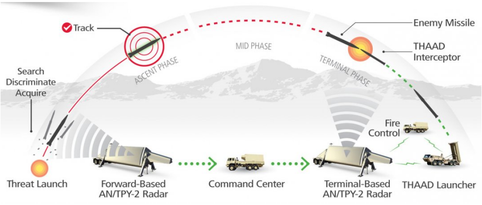

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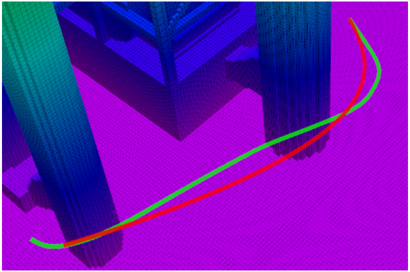
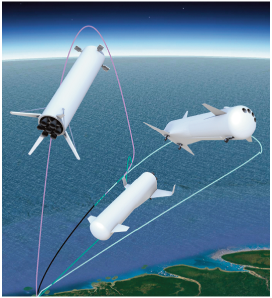
Motion Planning for Unmanned Aerial Vehicles

- **Objective** : Motion planning of Aerial Robots in cluttered environments.
- **Difficulty** : Dynamic environments (moving obstacles, etc.), obstacle modeling.



Reusable Launch Vehicles

- **Objective** : Return and landing of the first stage of space launchers.
- **Difficulty** : Highly constrained problem (aerodynamic and safety constraints), may need re-ignition of rocket engines.



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Real-time Optimal Control

Challenge

Compute optimal trajectories in **real time**, by using **embedded computers**, to make the vehicle adapt its trajectory to **changes of the scenario**.

- Global approaches (e.g. HJB):

Pros:

- Global optimum
- Do not require any initial guess

Cons:

- Time consuming for problems of high dimension
- Cannot be used for real time computations

- **Local approaches**

Real-time Optimal Control

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Compute optimal trajectories in **real time**, by using **embedded computers**, to make the vehicle adapt its trajectory to **changes of the scenario**.

1. Explicit Feedback Laws and Direct Methods (e.g. SQP):

Pros:
- Easy to implement
- Robustness

Cons:
- Lack of precision
- Expensive or sub-optimal

2. **Indirect Methods:**

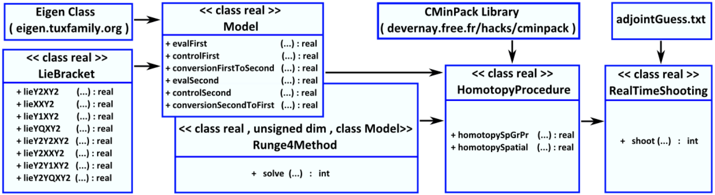
Pros:
- High precision
- **Fast convergence**

Cons:
- Complex analysis
- **Hard to initialize**

A library for solving real-time optimal control problems

SOCP (Shooting for Optimal Control Problems)

- **Indirect methods** for high precision.



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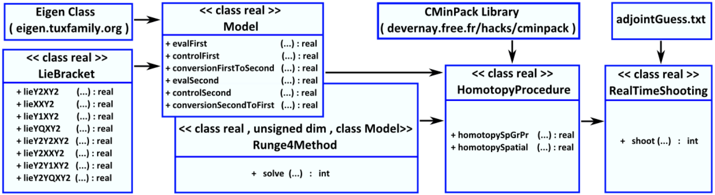
A library for solving real-time optimal control problems

SOCP (Shooting for Optimal Control Problems)

- **Indirect methods** for high precision.
- **Multiple shooting** for numerical robustness.



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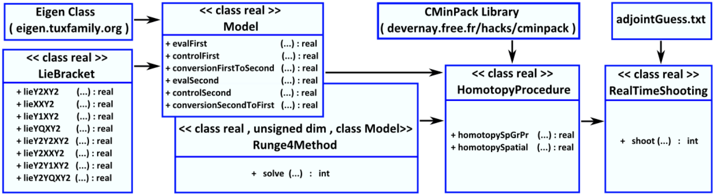

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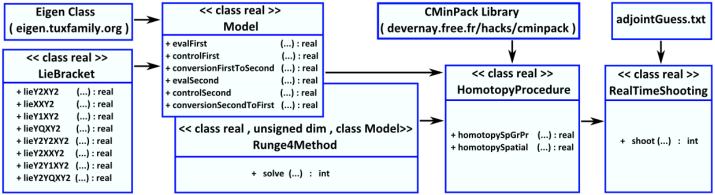

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A library for solving real-time optimal control problems

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- **Indirect methods** for high precision.
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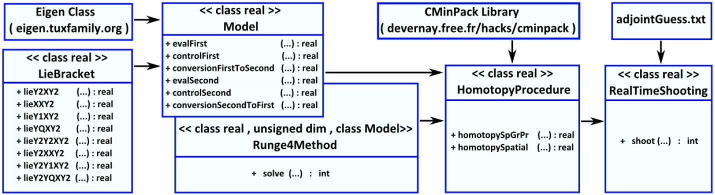
- **Homotopy methods** for initialization problems.
- **Parallel computing** for computation time.
- **C++ library** for best performance and embedded solutions.



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Return trajectory of Reusable Launch Vehicles

Objective (from the stage separation to the landing maneuver)

- Trajectory minimizing propellant consumption.
- Fixed final position and final velocity, free final time.
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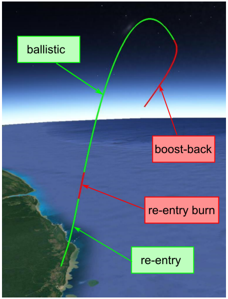

Figure: Toss-back model

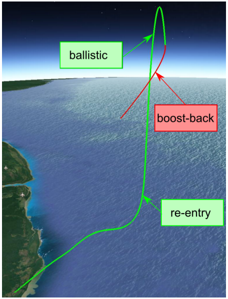

Figure: Glider model

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RLV model

$$
\begin{cases}
\min \quad -m(t_f) \\
\dot{\boldsymbol{r}} = \boldsymbol{v} \quad , \quad (\boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) \in \mathbb{R}^9 \quad , \quad \|r\| = 1 \\
\dot{\boldsymbol{v}} = \frac{L}{m}\boldsymbol{k}_a - \frac{D}{m}\frac{\boldsymbol{v}_a}{\|\boldsymbol{v}_a\|} - \boldsymbol{g} + \frac{T_m}{m}\gamma \boldsymbol{u} - 2\Omega \times \boldsymbol{v} - \Omega \times (\Omega \times \boldsymbol{r}) \\
\dot{m} = -q_m \gamma \quad , \quad 0 \leq \gamma \leq 1 \\
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Optimal Control with pure state constraints and mixed constraints

Optimal Control Problem

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\quad \min \quad \int_0^{t_f} f^0(X,U,t)dt + g(X(t_f),tf) \\
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X(x) = X_0, \boldsymbol{r}(t_f) = \boldsymbol{r}_f \\
t_f, m(t_f) \text{ and } \boldsymbol{v}(t_f) \text{ are free,} \\
\forall t, c_p(X(t)) \leq 0, \\
\forall t, c_m(X(t),U(t)) \leq 0.
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where $X = (\boldsymbol{r}, \boldsymbol{v}, m)$ and $U = (u, \gamma)$

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Pontryagin's Maximum Principle

$p = (p_r, p_v, p_m) \in \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}$ denotes the co-state vector

Hamiltonians

The extended Hamiltonian including the constraints

$$\tilde{H} = H + \alpha_p c_p(X) + \alpha_m c_m(X, U)$$

where the Hamiltonian $H = p.f(X, U, t) + p^0 f^0(X, u, t)$

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$$\dot{p} = -\frac{\partial \tilde{H}}{\partial X}$$

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Minimization condition

$$H(t, X, p, p^0, U) = \min_{V \in \mathcal{U}} H(t, X, p, p^0, V)$$

- If f and f^0 do not explicitly depend on t, H **is constant**.

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Transversality conditions

Since $m(t_f)$ and $\boldsymbol{v}(t_f)$ are free,

$$\begin{cases} p_v(t_f) = p^0 \frac{\partial g}{\partial v}(t_f, X(t_f)) \\ p_m(t_f) = p^0 \frac{\partial g}{\partial m}(t_f, X(t_f)) \end{cases}$$

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Continuity / Jump conditions

c_m is assumed regular (**MF-CQ**).

- α_p and α_p are continuous and $\forall t, \alpha_p c_p(X) = \alpha_m c_m(X, U) = 0$

At every contact point t_i with the boundary of the pure state constraint,

- $\tilde{H}(t_i^+) = \tilde{H}(t_i^-)$ and $\exists \nu_i, p(t_i^+) = p(t_i^-) - \nu_i \dfrac{\partial c_p}{\partial X}(X(t_i))$.

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Structure of the control

$J(\gamma, u)$ denotes the expression in the Hamiltonian that depends on the control:

$$H = J(\gamma, u) + \ldots$$

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- **Hypothesis: Aerodynamic forces are assumed to be negligible when the vehicle is in high altitude**

$$\Longrightarrow J(\gamma, u) = \gamma \underbrace{\left(1 - q_m p_m + \frac{T_m}{m} \langle p_v, u \rangle\right)}_{\psi}$$

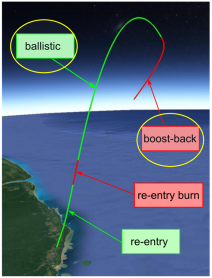

Figure: Boost-Back and ballistic phases

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Minimization condition

$$\Longrightarrow u = -\frac{\boldsymbol{p}_v}{\|\boldsymbol{p}_v\|}$$

- When the switching function $\Psi < 0$, $\gamma = 1$.
- When the switching function $\Psi > 0$, $\gamma = 0$.

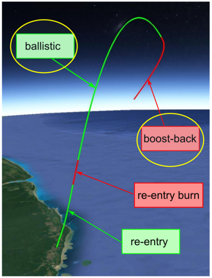
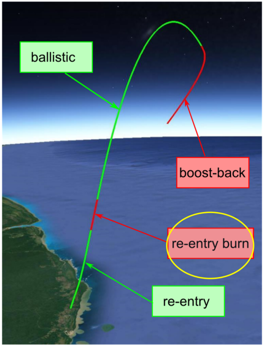


Figure: Boost-Back and ballistic phases

Re-entry burn

The previous hypothesis is irrelevant during the re-entry burn. It may complicate the explicit computation of the optimal control input.



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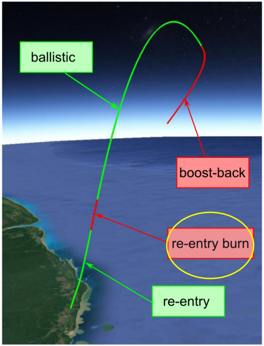
- **Hypothesis**: **Aerodynamic forces remain low during this phase**

$$\Longrightarrow u = -\frac{p_v}{\|p_v\|}$$



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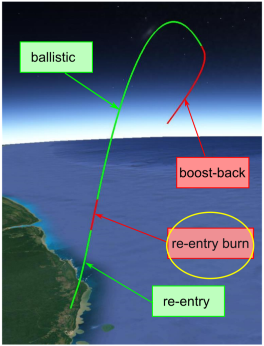
$\Longrightarrow$ γ can be mathematically complex.



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Solution

- A constant approximation is taken for the value of γ along this phase (e.g. $\gamma = 1/3$).

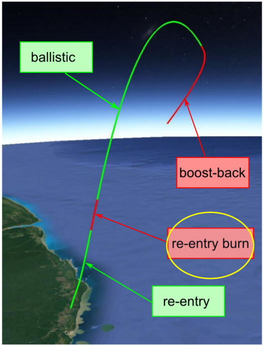


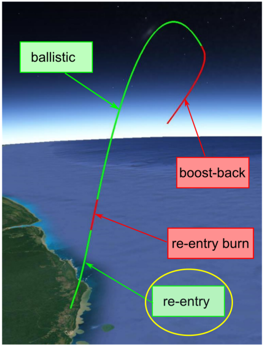
Figure: Re-entry burn

Atmospheric re-entry

- Ballistic phase, i.e. $\gamma = 0$
- The aerodynamic forces are considered.
- The expression depending on the control reads

$$J = \frac{L}{m} \langle \boldsymbol{u}, \boldsymbol{v} \rangle \langle \boldsymbol{u}, p_v \rangle$$

- $\boldsymbol{u}$ which minimizes J **can be explicitly computed**.



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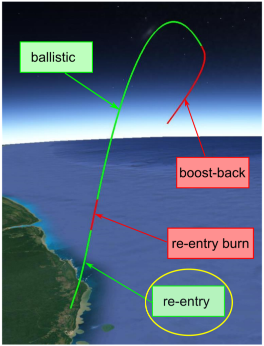

Figure: Atmospheric re-entry phase

More details:
E. Brendel, B. Hérissé and E. Bourgeois, "**Optimal guidance for Toss Back concepts of Reusable Launch Vehicles**", 8th European Conference for Aeronautics AND Aerospace Sciences (EUCASS), 2019.

Homotopy method

- Solve a simplified problem for which an explicit solution can be found.

RLV model

$$
\begin{cases}
\min \quad - \quad m(t_f) \\
\dot{\boldsymbol{r}} = \boldsymbol{v} \quad , \quad (\boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) \in \mathbb{R}^9, \quad \|\boldsymbol{r}\| = 1 \\
\dot{\boldsymbol{v}} = \frac{L}{m}\boldsymbol{k}_a - \frac{D}{m}\frac{\boldsymbol{v}_a}{\|\boldsymbol{v}_a\|} - \boldsymbol{g} + \frac{T_m}{m}\gamma\boldsymbol{u} - 2\Omega \times \boldsymbol{v} - \Omega \times (\Omega \times \boldsymbol{r}) \\
\dot{m} = - \quad q_m\gamma \quad , \quad 0 \le \gamma \le 1 \\
(\boldsymbol{r}, \boldsymbol{v}, m)(0) = (\boldsymbol{r}_0, \boldsymbol{v}_0, m_0) \ , \ (\boldsymbol{r}, \boldsymbol{v})(t_f) = (\boldsymbol{r}_f, \boldsymbol{v}_f) \\
\alpha \le \alpha_{\max} \quad \text{if} \quad \|r\| \le r_{\text{ref}} \ , \quad Q \le Q_{\max} \ , \quad \Phi \le \Phi_{\max} \ , \quad \eta_x \le \eta_{x,\max} \ , \quad \eta_z \le \eta_{z,\max}
\end{cases}
$$

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Homotopy method

- Solve a simplified problem for which an explicit solution can be found.

RLV model (homotopy start with $\mu_x = 0$ except $\mu_{L_2} = 1$)

$$\begin{cases} \min \quad -\mu_{L_1} m(t_f) + \mu_{L_2} \int_0^{t_f} \gamma^2 \, dt \\ \dot{\boldsymbol{r}} = \boldsymbol{v} \quad , \quad (\boldsymbol{r}, \boldsymbol{v}, \boldsymbol{u}) \in \mathbb{R}^9, \quad \|r\| = 1 \\ \dot{\boldsymbol{v}} = \mu_L \frac{L}{m} \boldsymbol{k}_a - \mu_D \frac{D}{m} \frac{\boldsymbol{v}_a}{\|\boldsymbol{v}_a\|} - \mu_g \boldsymbol{g} + \frac{T_m}{m} \gamma \boldsymbol{u} - 2\Omega \times \boldsymbol{v} - \Omega \times (\Omega \times \boldsymbol{r}) \\ \dot{m} = -\mu_q q_m \gamma \quad , \quad 0 \leq \gamma \leq 1 \\ (\boldsymbol{r}, \boldsymbol{v}, m)(0) = (\boldsymbol{r}_0, \boldsymbol{v}_0, m_0) \ , \ (\boldsymbol{r}, \boldsymbol{v})(t_f) = (\boldsymbol{r}_f, \boldsymbol{v}_f) \\ \mu_\alpha \alpha \leq \alpha_{\max} \quad \text{if} \quad \|r\| \leq r_{\text{ref}} \ , \ \mu_Q Q \leq Q_{\max} \ , \ \mu_\Phi \Phi \leq \Phi_{\max} \ , \ \mu_\eta^x \eta_x \leq \eta_{x,\max} \ , \ \mu_\eta^z \eta_z \leq \eta_{z,\max} \end{cases}$$

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Homotopy method

- Solve a simplified problem for which an explicit solution can be found.

  ↖ Here, a double integrator!

RLV model (homotopy start with $\mu_x = 0$ except $\mu_{L_2} = 1$)

$$
\begin{cases}
\min \quad -\mu_{L_1} m(t_f) + \mu_{L_2} \int_0^{t_f} \gamma^2 \, dt \\
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$$

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Homotopy method

- Solve a simplified problem for which an explicit solution can be found.
  ↖ Here, a double integrator!
- Initialize the co-state vector of a multiple shooting algorithm.

RLV model (homotopy start with $\mu_x = 0$ except $\mu_{L_2} = 1$)

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Homotopy method

- Solve a simplified problem for which an explicit solution can be found.
  ↖ Here, a double integrator!
- Initialize the co-state vector of a multiple shooting algorithm.
- Then, every neglected term is gradually added until the problem being solved is the complete problem.

RLV model (homotopy start with $\mu_x = 0$ except $\mu_{L_2} = 1$)

$$
\begin{cases}
\min \quad -\mu_{L_1} m(t_f) + \mu_{L_2} \int_0^{t_f} \gamma^2 \, dt \\
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\end{cases}
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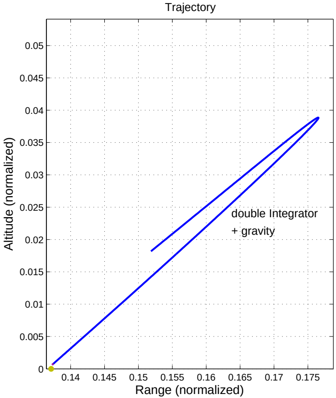
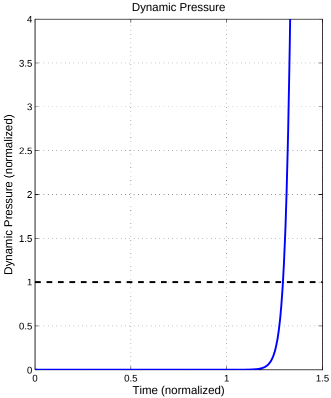
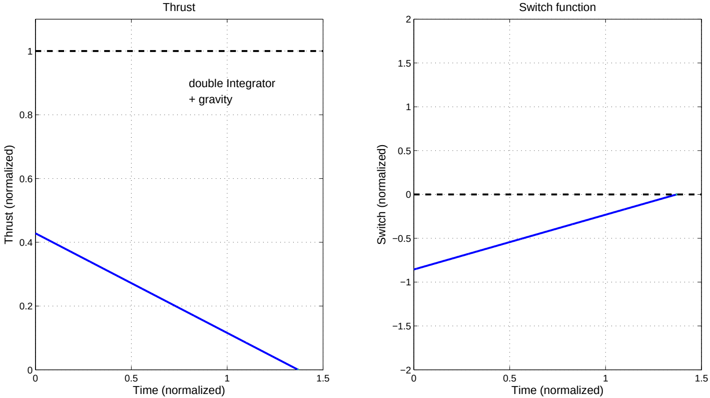
Numerical results: Toss-back model





Numerical results: Toss-back model



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Optimal Guidance algorithm

- The **offline** reference trajectory is used to initialize the guidance algorithm
- Recompute the optimal control **online** with an homotopy on the initial state
- **Evaluation**: with randomly scattered parameters, e.g. **initial state dispersion**, aerodynamic coefficients, maximal thrust, maximal mass flow rate...

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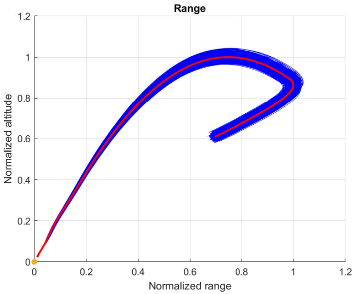

Satisfactory precision of the guidance algorithm: The margin of error on final position is below 100m (at 3km of the landing plateform)

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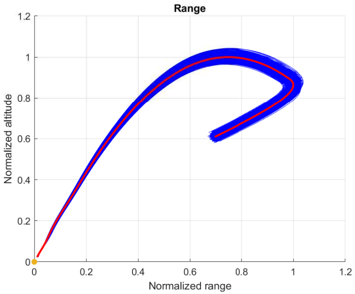

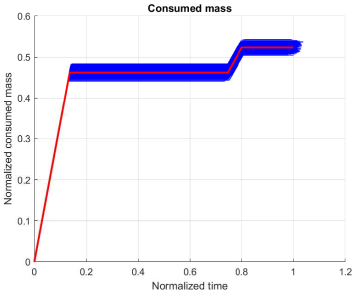

Satisfactory precision of the guidance algorithm: The margin of error on final position is below 100m (at 3km of the landing plateform)

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Optimal Guidance algorithm

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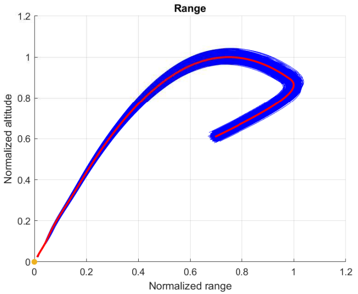

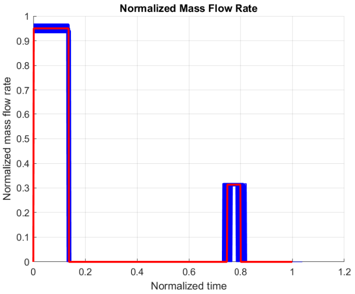

Satisfactory precision of the guidance algorithm: The margin of error on final position is below 100m (at 3km of the landing plateform)

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Outline

Toss-back vs Glider

- Similar models
- Vertical Takeoff Horizontal Landing
- The glider model uses **a larger L/D ratio**.

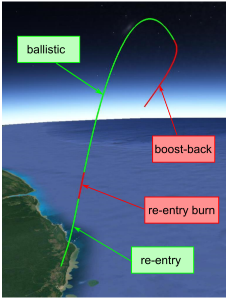


Figure: Tossback recovery

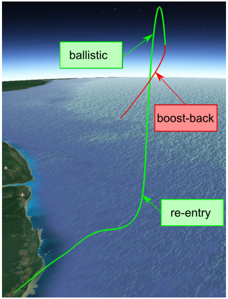
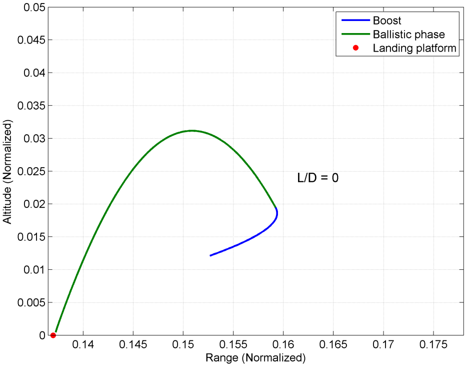


Figure: Glider model

Numerical results: Glider model

- Homotopy on the L/D ratio



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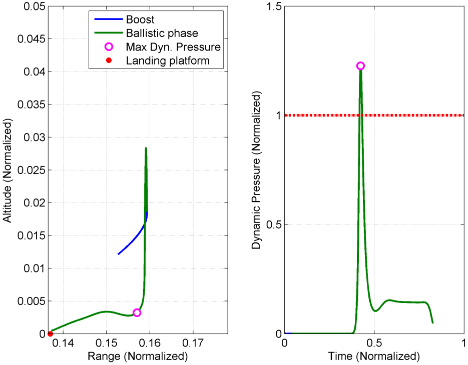
Numerical results: Glider model

- Homotopy on the the dynamic pressure



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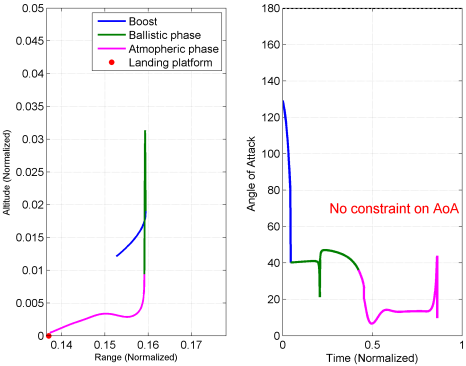
Numerical results: Glider model

- Saturation of the angle-of-attack



Outline

Conclusion

Conclusion

- An efficient **C++ library** implementing shooting methods.
- A general homotopy scheme for solving RLVs problems.
- The algorithm can be adapted for on-line recalculations with a more realistic atmospheric model.

Perspectives

- Complete the study of the glider model (the load factor constraint, the optimal guidance,etc.)
- Compare with direct methods.
- Assess the flight-back approach.

The SOCP Team

https://github.com/bherisse/socp

Prince EDORH
DTIS/NGPA

Elliot BRENDEL
DTIS/NGPA

Bruno HÉRISSÉ
DTIS/NGPA

Riccardo BONALLI
DTIS/NGPA alumni